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The Role of Recycle in Countercurrent Recycle Distillation Cascades. I. Constant Reflux, Ideal, and Squared-Off Cascades

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ABSTRACT

In most textbooks concerned with countercurrent multistage separations, minimum reflux ratio for continuous distillation is usually defined only in terms of a graphical construction on a McCabe–Thiele diagram: it is the recycle ratio (liquid flow rate) associated with the operating line that touches the equilibrium curve *at the feed point*. However, it is easily shown that minimum recycle ratio depends on local α and composition, as well as product compositions, and thus, it is a stage-composition phenomenon. As a result, for a specified separation, each ideal stage in a continuous distillation cascade has a specific minimum recycle ratio associated with it. For constant α, the minimum recycle ratio increases as the stage compositions depart more from product (distillate or bottoms) compositions. As a result, the textbooks only consider the *maximum* minimum recycle ratio. This paper presents the results of some theoretical calculations which illustrate how minimum recycle ratio varies with stage α, stage and product compositions, and presents an example of distillation cascade behavior when minimum recycle ratio is approached at a composition other than the feed point. An example is also presented which shows how the separation is effected when the reflux ratio is reduced below the design value in a distillation column containing a fixed number of ideal stages. A brief comparison is also made between constant reflux, ideal, and squared-off cascades in terms of number of stages, total interstage flow, and relative energy requirements for the different designs to illustrate and emphasize the consequences of the stagewise behavior of minimum recycle ratio.

INTRODUCTION AND BACKGROUND

If one reads about minimum reflux for continuous distillation in the most recent textbooks on staged separations, it is found that minimum reflux ratio [$(R_i)_{min}$] (or RR_{min}) is only defined in terms of where an "operating line" touches the equilibrium curve *at the feed point* on a McCabe–Thiele diagram. The only other case considered is for an equilibrium point tangent to the operating line for nonideal systems (1–5). Thus, in the textbooks, $(R_i)_{min}$ is usually determined only in terms of a graphical construction on a McCabe–Thiele diagram, i.e., a diagram which includes an equilibrium (x, y) curve together with a single straight operating (material balance) line that connects the product composition on the $x = y$ line, to the feed composition on the equilibrium curve. It is then seen from this graphical construction that, in the limit as minimum flow conditions are approached, i.e., as the operating line that produces $(R_i)_{min}$ is approached, an infinite number of stages would be required to transgress the feed stage composition. However, as shown by Benedict (6) nearly half a century ago, $(R_i)_{min}$ is a function of y_P, x_B, and local α and local (stage) composition and thus, since local composition varies with each stage (local α can also vary), each stage in a separation cascade has a specific minimum recycle ratio associated with it. For constant α, the minimum recycle ratio increases as the composition within the separation cascade departs more-and-more from the desired product compositions, and so is a maximum at the feed stage. Thus, the textbooks only consider the *Maximum* $(R_i)_{min}$, and a reflux ratio somewhat greater than this $[(R_i)_{min}]_{max}$ is used for the design criteria throughout the distillation column (cascade). However, as a result of the dependence of $(R_i)_{min}$ on stage α, stage, and desired product compositions, it is theoretically possible to design a separation cascade that has a different recycle ratio at each stage, or one that contains several constant recycle sections in it, provided that each stage recycle ratio as determined by the composition and α on the stages is greater than the respective minimum values.

The textbook approach of considering only $[(R_i)_{min}]_{max}$ may be adequate for the engineering design and analysis of ordinary distillation columns but, in general, it is too restrictive. The textbooks only tell part of the recycle ratio story; very little is learned about the basic principles behind the need for reflux in multistage separations. We only learn that a flow rate above a certain minimum is required to transgress the pinch point at the feed composition. There may be systems and situations where the use of varying recycle ratio in a distillation cascade may be economically attractive. In any case, the consideration of only $[(R_i)_{min}]_{max}$ in the discussion and teaching of separation science does little to further it.

This paper reintroduces some of the basic concepts for multistage separation processes as presented by Benedict (6, 7), with emphasis on the compositional and relative volatility/stage behavior of $(R_i)_{min}$, and suggests areas where these basic principles may possibly be used to advantage to design more efficient distillation processes.

Review of Some Basic Separation Concepts

As discussed by Pfann (8), the act of separation requires that a concentration difference be established between two regions. Different separation processes have evolved by taking advantage of various physical and/or chemical phenomena to establish this concentration difference between regions. For the process of distillation, the phenomenon is vapor–liquid (phase) equilibrium. Only the simple case of the separation of binary mixtures will be considered.

Benedict (6) has very nicely discussed the basic separation principles for binary multistage separations, and his treatise will be followed here to serve as a basis for the developments and discussions that follow. Although distillation is specifically discussed in the following sections, most of the basic ideas apply to any other separation processes which can be carried out in countercurrent recycle cascades.

It is usually convenient to carry out engineering analysis of separation processes in terms of discrete separation stages. This is frequently done even though actual physical flow and contacting in a real separation cascade may properly be termed "differential," i.e., a packed distillation column where a stage is defined in terms of packed column height to give product compositions equivalent to a discrete stage. The elementary separation stage can be defined in a number of ways, but the simplest stage in a continuous process is one that receives one or more feed streams, and produces a *heads* stream that is enriched in the desired product, and a depleted *tails* stream. In distillation the *ideal stage* is usually defined as a stage where the heads and tails streams leaving the stage are in vapor–liquid (phase) equilibrium. For this case the heads and tails stream compositions are related by the *stage separation factor*, α_i, via the equation

$$\frac{y_i}{1 - y_i} = \alpha_i \frac{x_i}{1 - x_i} \tag{1}$$

Here, y_i and x_i are the compositions leaving stage i, and α_i is the *relative volatility* which is a specific example of a stage separation factor.

Usually the desired overall separation is greater than what can be accomplished in one stage, and so it is necessary to employ a number of stages connected in series to obtain the desired separation. The interconnected series of stages is usually referred to as a *cascade*. There can be many cascade configurations, and apparently there is no natural law that can be used to predict the best flow configuration for a given system. Based on experience however, countercurrent recycle cascades (CRCs) have proven to be effective in carrying out many separations. The recycle flow pattern in ordinary continuous distillation (in terms of discrete stages) is a special case of a CRC. A schematic diagram of a discrete stage distillation column is shown in Fig. 1.

The distillation CRC consists of a number (n) of ideal stages connected in series, each receiving two feed streams: down-flowing liquid from the next higher stage, and up-flowing vapor from the next lower stage. The stages produce stage heads streams (rate M_i, composition y_i) and tails streams (rate N_i, composition x_i). For the ideal stage the heads and tails streams leaving the stage are in equilibrium, that is, x_i and y_i are related

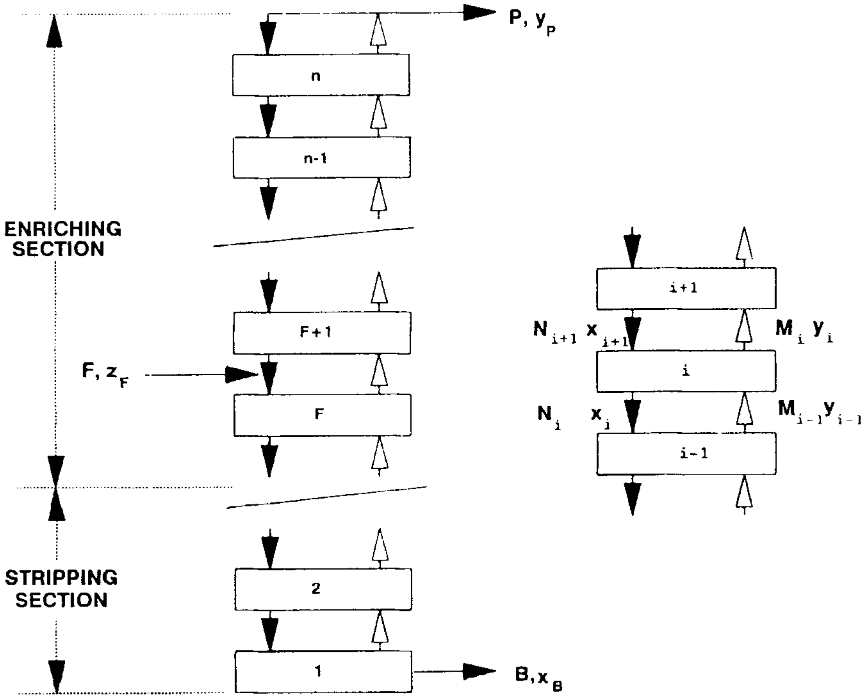


FIG. 1 Schematic diagram of a countercurrent recycle distillation cascade.

by Eq. (1). If *designed correctly* (with respect to flow rates, number of stages, etc.), the overall cascade will separate a feed stream, rate F, composition z_F, into a product stream rate P, composition y_P, and a bottoms stream, rate B, composition x_B. The portion of the cascade from (and including) the feed stage up to the product stage is referred to as the *enriching section*, while that portion below the feed stage is the *stripping section*.

As shown in Fig. 1, the tails from stage $i + 1$ is recycled to become part of the feed to stage i, with the other part of the feed being the heads from stage $i - 1$; while the heads stream from stage i and the tails stream from stage $i + 2$ forms the feed to stage $i + 1$, etc. In distillation, intimate mixing of the heads and tails streams making up the feed to a stage is necessary in order for the stage to make a separation: the assumption that each feed stream is individually separated into equilibrium stage heads and tails streams *cannot* be consistent with mass and energy balance requirements around individual stages. Much of the material in the feed heads stream must end up in the stage tails stream, and vice versa. The required flow pattern is produced by the internal hardware inside an ordinary plate distillation column, and this specific flow pattern is what characterizes a simple countercurrent recycle cascade. As shown below, this stage recycle (reflux) is necessary for the cascade to make the desired overall separation, and thus, its rate is an important design variable. It is convenient to characterize the flow of recycle tails stream to a stage in terms of a stage recycle ratio, which is defined as

$$R_i = N_{i+1}/P \tag{2}$$

Note that R_i is defined in terms of N_{i+1}, the tails rate from the stage above stage i (i.e., stage $i + 1$) that becomes part of the feed to stage i.

The degree of enrichment effected by a single stage of a cascade depends on stage separation factor, net flow through the stage, interstage flow [i.e., (R_i)], and local (stage) compositions.

The cascade must be designed so that, during steady-state operation, the withdrawal of top and bottom products is maintained. Thus, there must be a net up-flow of $(y_P)(P)$ moles of the more volatile component through each stage in the enriching section, and a net down-flow of $(x_B)(B)$ moles of the heavier component through each stage in the stripping section. Material balances around enriching and stripping sections to stage i in the enriching section and to stage j in the stripping section expressing these requirements are given by

$$\text{Enriching section:} \qquad y_i - x_{i+1} = (y_P - y_i)\frac{P}{N_{i+1}} \tag{3}$$

$$\text{Stripping section:} \qquad y_j - x_{j+1} = (y_j - x_B)\frac{B}{N_{j+1}} \tag{4}$$

Here, j is used to designate a general stage in the stripping section since different equations result from material balances in the different sections. The difference in composition between tails streams leaving *adjacent* stages is conveniently obtained by combining Eqs. (3) or (4) with the *enrichment factor*, $(\alpha_i - 1)$, which is obtained by rearranging Eq. (1). These differences, which characterize the separation that is occurring on stage i or j, are

$$\text{Enriching section} \qquad x_{i+1} - x_i = (\alpha_i - 1)(x_i)(1 - y_i) - (y_P - y_i)\frac{P}{N_{i+1}} \tag{5}$$

$$\text{Stripping section} \qquad x_{j+1} - x_j = (\alpha_j - 1)(x_j)(1 - y_j) - (y_j - x_B)\frac{B}{N_{j+1}} \tag{6}$$

As can be seen from these equations, the depletion of the tails streams leaving adjacent stages in the cascade depends on the stage α, stage composition, and on P/N_{i+1} or B/N_{j+1} for specified values of y_P and x_B. As also can be seen, the separation occurring on a stage can be increase within limits by increasing N_{i+1}, since the negative term in Eqs. (5) and (6) becomes smaller. It is seen by calculating values of $(x_{i+1} - x_i)$ (for a specified set of parameters) as N_{i+1} is decreased more and more, that a minimum value of N_{i+1} will be reached where separation will cease to occur, that is, where $(x_{i+1} - x_i)$ is equal to zero. It is convenient to define these minimum flows in terms of recycle ratios. Setting $(x_{i+1} - x_i)$ and $(x_{j+1} - x_j)$ equal to zero in Eqs. (5) and (6), and rearranging, results in the following equations:

$$\text{Enriching section} \qquad \left[\frac{N_{i+1}}{P}\right]_{\min} = \frac{(y_P - y_i)[y_i + \alpha_i(1 - y_i)]}{(\alpha_i - 1)(y_i)(1 - y_i)} \tag{7}$$

$$\text{Stripping section} \qquad \left[\frac{N_{j+1}}{B}\right]_{\min} = \frac{\alpha_j x_j - x_B[1 + (\alpha_j - 1)x_j]}{(\alpha_j - 1)(x_j)(1 - x_j)} \tag{8}$$

Although not necessary, it is convenient to express $(R_i)_{\min}$ in terms of equilibrium liquid stage composition in the stripping section and equilibrium vapor stage composition in the enriching section. Many different equivalent forms of these equations are possible since x_i and y_i are related by Eq. (1), i.e., they are in equilibrium, and $F = B + P$ (see Appendix 1). The recycle ratios for both enriching and stripping sections can be put on the same basis by

$$\left[\frac{N_{j+1}}{P}\right]_{\min} = \left[\frac{N_{j+1}}{B}\right]_{\min}\left[\frac{B}{P}\right] = \left[\frac{N_{j+1}}{B}\right]_{\min}\left[\frac{F}{P} - 1\right] = \left[\frac{N_{j+1}}{B}\right]_{\min}\left[\frac{y_P - z_F}{z_F - x_B}\right] \tag{9}$$

These equations come from a consideration of material balance and stage equilibrium requirements around individual stages, and the entire enriching or stripping sections of a distillation cascade. Equations (7) and (8) result from applying the equilibrium/material balance requirements to *adjacent* stages as the liquid flow rate entering a stage from the stage above is reduced. Note that N_{i+1} and N_{j+1} are the liquid flow rates entering a stage from the stage above it; and that the stage compositions leaving the adjacent stages approach each other as N_{i+1} or N_{j+1} is reduced, thus requiring more and more stages to reach the "critical" composition. In the limit, $[N_{i+1}]_{\min}$ or $[N_{j+1}]_{\min}$ results in an infinite number of stages being required to reach the critical composition where separation ceases between adjacent stages, that is, where $x_i = x_{i+1}$.

The derivation of these equations shows that minimum recycle ratio is a *stagewise-composition* phenomenon. If interstage flow is maintained above $[N_{i+1}]_{\min}$, separation will occur on the stage, that is, $(x_{i+1} - x_i) > 0$. The separation will become less and less as $[N_{i+1}]_{\min}$ is approached, becoming zero at $[N_{i+1}])_{\min}$.

To emphasize the effects of N_{i+1} on the separation occurring on a stage with specified α_i and y_i, or x_i, it is convenient to use a simple example. Figure 2 depicts equilibrium stage *i* in the enriching section of a distillation cascade where (for illustration purposes) $\alpha_i = 4.0$ and $y_i = 0.5$. As a result of Eq. (1), $x_i = 0.2$. Consider the case where the column design results in $P = 1$ and $y_P = 0.8$. From Eq. (7), $[N_{i+1}]_{\min}$ and $(R_i)_{\min} = 1.0$.

Material balances around the enriching section of the cascade yields

$$M_i = N_{i+1} = P \tag{10}$$

$$x_{i+1} = \frac{[(M_i)(y_i) - (P)(y_P)]}{N_{i+1}} \tag{11}$$

$$y_{i-1} = \frac{[(N_i)(x_i) + (P)(y_P)]}{M_{i-1}} \tag{12}$$

As a result of these material balance requirements, x_{i+1} and y_{i-1} must vary with N_{i+1}, which can take on values from very large (i.e., approach-

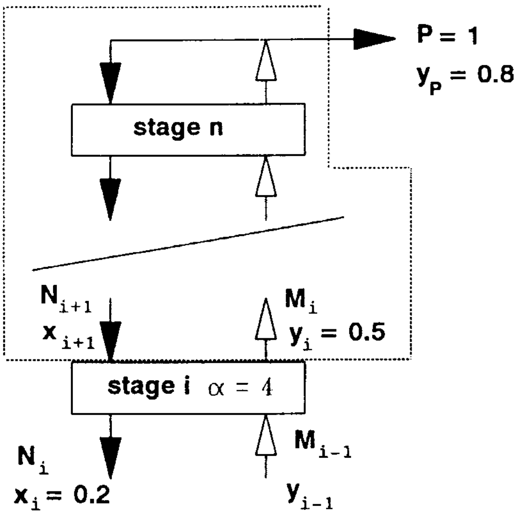


FIG. 2 Stage i in the enriching section of a distillation cascade with $\alpha = 4$, $y_i = 0.5$.

ing total reflux) to smaller values approaching $[N_{i+1}]_{min}$ and still maintain separation on that stage. The possible ranges of values for x_{i+1} and y_{i-1} as N_{i+1} varies between total and minimum recycle are shown in Table 1.

x_{i+1} and y_{i-1} must change with N_{i+1} to be consistent with the assumed external variables for the cascade which requires that there be a net up-flow of $(y_P)(P)$ in each stage in the enriching section of the cascade. The two limiting cases for total reflux ($N_{i+1} \rightarrow \infty$) and $N_{i+1} \rightarrow [N_{i+1}]_{min}$ are shown in Fig. 3.

The actual separation occurring on the stage [as measured by $(x_{i+1} - x_i)$] varies widely with R_i, with the maximum change in composition $[(x_{i+1} - x_i) = (0.5 - 0.2) = 0.3]$ taking place as total reflux is approached. The stage separation becomes less and less as N_{i+1} is reduced, and becomes zero $[(x_{i+1} - x_i) = (0.2 - 0.2) = 0]$ at $[N_{i+1}]_{min}$. For values of N_{i+1} less than $[N_{i+1}]_{min}$, material balances would require that the more volatile compound be concentrated in the tails stream, which is not consistent with the requirements for a stage with $\alpha > 1$.

For these calculations it is immaterial what happens between stage i and stage N as long as a net up-flow of $(y_P)(P)$ is maintained throughout that section of the cascade; but (vide infra) as $[N_{i+1}]_{min}$ is approached,

TABLE I
x_{i+1} and y_{i-1} as a Function of N_{i+1} for Stage i shown in Fig. 2. Calculation of y_{i-1} Assumes Constant Molal Overflow. Note that N_{i+1} and $(R_i)_{min}$ = 1 for the Assumed Conditions

N_{i+1}	x_{i+1}	y_{i-1}
∞	0.50000	0.20000
10,000	0.49997	0.20006
100	0.49700	0.20594
10	0.47000	0.25455
5	0.44000	0.30000
2	0.35000	0.40000
1.5	0.30000	0.44000
1.2	0.25000	0.47273
1.1	0.22727	0.48571
1.001	0.20030	0.49984
1.0 = $[N_{i+1}]_{min}$	0.20000	0.50000

less and less separation will also occur in the stages adjacent and above stage i, and more and more stages are required to reach the local composition associated with that value of $[N_{i+1}]_{min}$. The dependence of $(R_i)_{min}$ on x_B, y_P, and local α and stage compositions can conveniently be illustrated on McCabe–Thiele-type diagrams. Plots showing the results of calculations for $(R_i)_{min}$ for specific values of local α and composition, x_B, and y_P are presented and discussed in the next section.

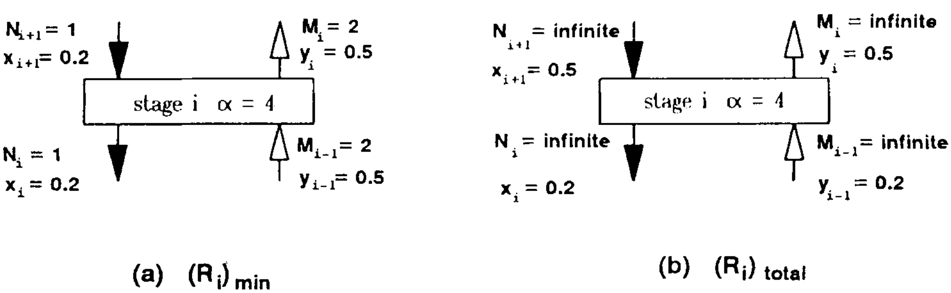


FIG. 3 Separation occurring on stage i of Fig. 2 for (a) minimum recycle, and (b) total reflux. Values assume constant molal overflow.

CALCULATIONS, RESULTS, AND DISCUSSION

General Dependence of $(R_i)_{min}$ on Cascade Variables

The trends for the dependence of $(R_i)_{min}$ on x_B, y_P, and local α and stage composition can conveniently be illustrated on McCabe–Thiele plots, and plots of $(R_i)_{min}$ as a function of x_i, y_i, and α for specified separations (i.e., specified z_F, x_B, and y_P).

Figure 4(a) shows the McCabe–Thiele construction for $(R_i)_{min}$ (which is a function of the slope of the operating line) in the enriching section for $y_i = 0.7$, $y_P = 0.9$ for $\alpha = 4.0$ and 1.5. These constructions result in $(R_i)_{min} = 0.60317$ and 2.19048 for $\alpha = 4$ and 1.5, respectively. Figure 4(b) shows the same information for $\alpha = 4$ and stage composition $y_i = 0.7$ and 0.5 when the specification for y_P is changed from 0.9 to 0.8. As can be seen, for constant $y_i = 0.7$, $(R_i)_{min} = 0.60317$ and 0.30159 for $y_P = 0.9$ and 0.8, respectively, while $(R_i)_{min} = 1.0000$ if $\alpha = 4$, $y_i = 0.5$, and $y_P = 0.8$, as shown in the previous example. Figure 4(c) shows an arbitrary McCabe–Thiele construction for a nonideal system where α is a function of composition. Figure 4(d) is a plot of α and $(R_i)_{min}$ as a function of x_i or y_i with $(R_i)_{min}$ shown for the case where $z_F = 0.5$, $x_B = 0.01$, and $y_P = 0.99$ for the system shown in Fig. 4(c). For this system, $(R_i)_{min}$ goes through a local minimum in the enriching section at $y_i \approx 0.7$ but increases as $y_i \rightarrow y_P$ because of the rapid decrease in α for higher stage concentrations.

Figure 5 presents $(R_i)_{min}$ as a function of α for some selected specified separations (i.e., specified z_F, x_B, and y_P) to further show how $(R_i)_{min}$ varies with the different independent variables. In these plots $[N_{j+1}/B]_{min}$ is multiplied by $[F/P - 1]$ to put the $(R_i)_{min}$ for the stripping and enriching sections on the same basis. These plots clearly show how $(R_i)_{min}$ depends on α, feed, and product compositions.

As can be seen from these figures, $(R_i)_{min}$ is a maximum at the feed composition, and decreases to smaller values as y_P and x_B are approached. $(R_i)_{min}$ is greater for smaller values of α, and goes through a rather sharp maximum at the feed composition as α approaches unity. As α is increased, the maximum at the feed composition becomes less pronounced. Higher $(R_i)_{min}$ are required for more concentrated products, and for feed streams that are less concentrated.

Figure 6 shows how the number of stages increases above stage i as $[N_{i+1}]_{min}$ is approached in a enriching section of a distillation column for a system with $\alpha = 2.0$. For this example, a cascade design is assumed where stage number 12 (arbitrary) is both a vapor–liquid contacting device and a partial condenser. The heads (vapor) stream from stage 11 is mixed with the tails stream from stage 13 and, in addition, there is energy removal at stage 12 so that net condensation takes place, with the result that the equilibrium heads stream rate leaving stage 12 is decreased and the stage 12 tails stream rate is increased. For a constant P rate, the R_i in the section above the partial condenser must decrease relative to the constant R_i below it. Figure 6 shows composition vs stage number for a sequence of cascade designs when more and more partial condensation takes place on stage 12, keeping the same R_i below that point. As can be seen, as $(R_i)_{min}$ is approached (as determined by the equilibrium composition on stage 12), more and more new stages must be included in the cascade design to reach the composition on stage 12, with the number of new stages becoming infinite in the limit as $(R_i)_{min}$ is reached.

Similar calculations could be made where R_i is reduced at the feed stage (where $x_{F+1} = z_F$), where for constant α, $(R_i)_{min}$ is a maximum.

Cascade Design Dependence on $(R_i)_{min}$

The various distillation cascades that are discussed below were modeled using a computer spreadsheet program by solving for stage compositions, number of stages, etc. using appropriate equilibrium and material balance equations; starting at the feed stage and working toward both product ends of the cascade. For these calculations the feed stage is assumed to receive a tails stream from stage $F + 1$ which has exactly the feed composition, and the feed (which is assumed saturated) is mixed with N_{F+1} before entering stage F. Cascade designs were limited to a discrete number of stages, and thus, some trial and error was necessary. As a result, product compositions varied slightly for given sets of calculations. Notwithstanding the small variations and inconsistencies at stage 1 and n due to the assumption of a discrete number of stages, all cascades performed essentially the overall separations specified for that particular study (vide infra). The calculations very nicely show the theoretical consequences of the composition-stage dependence of recycle ratio.

Constant Reflux, Ideal, and Squared-Off Cascades

It should be noted that, in the discussions that follow (as in previous discussions), i is sometimes used to designate a general stage in either stripping or enriching sections of a cascade, for convenience.

It is apparent from the previous discussion that any desired stage recycle ratio can theoretically be maintained by the use of a partial condenser or reboiler at that stage. Hence a cascade can be designed to maintain any desired recycle ratio profile in it by use of partial condensers or reboilers at appropriate points within the cascade. This could range from the use of a single reboiler and condenser for a constant recycle design, to the

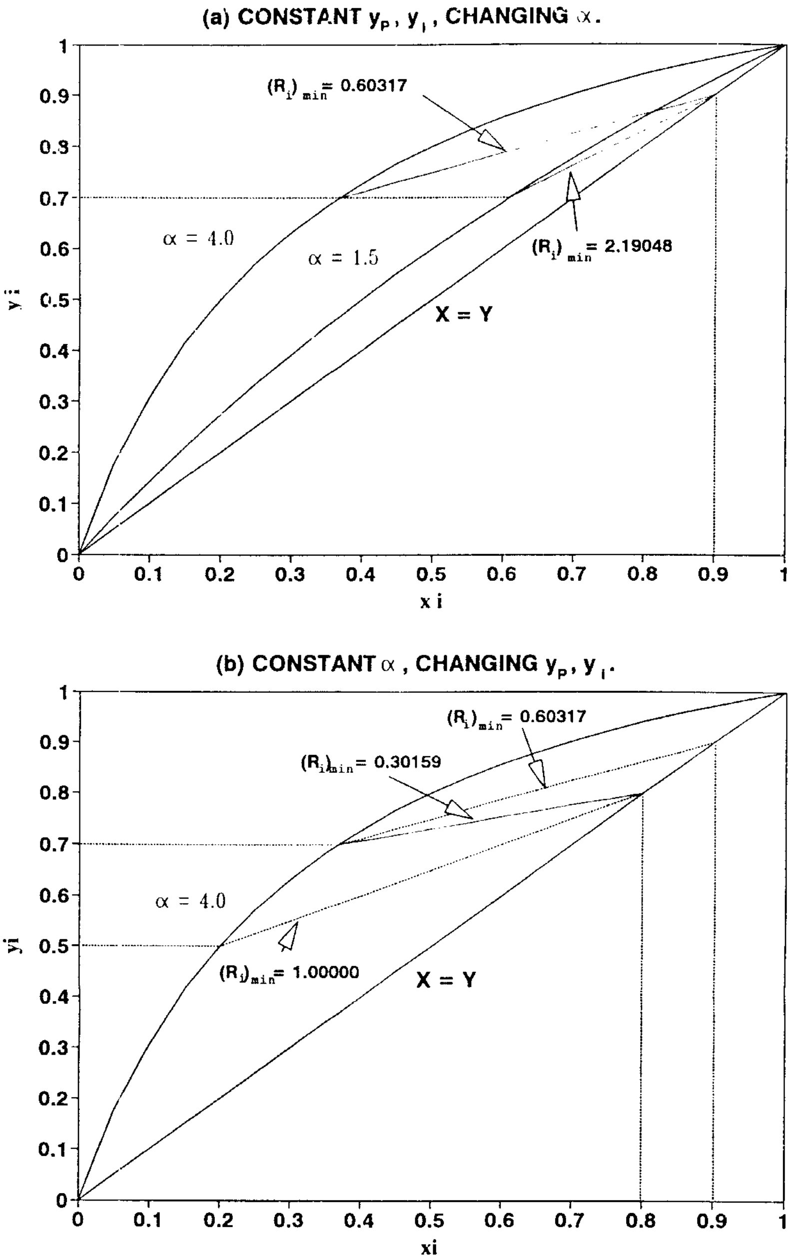


FIG. 4 McCabe–Thiele illustrations for minimum recycle ratios for various arbitrary combinations of stage compositions, stage α, and product compositions.

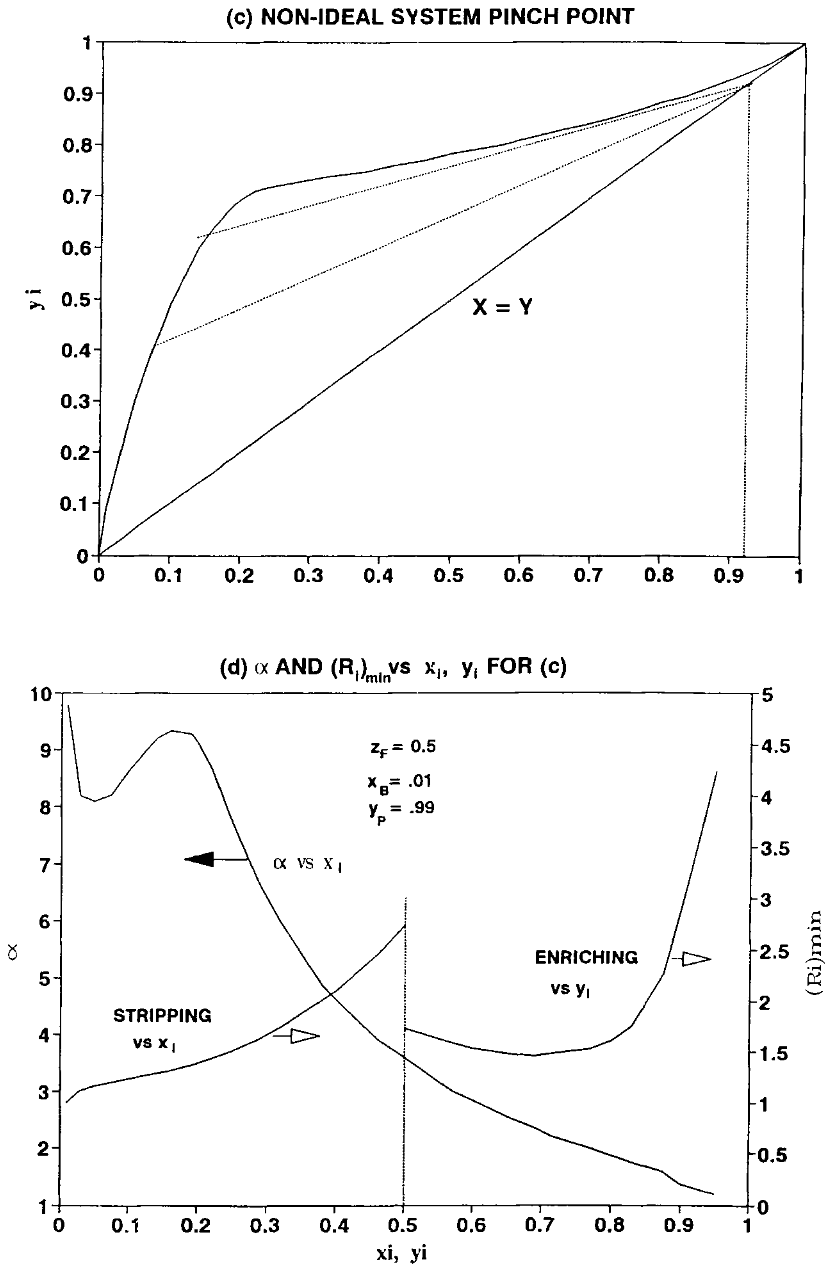


FIG. 4 Continued

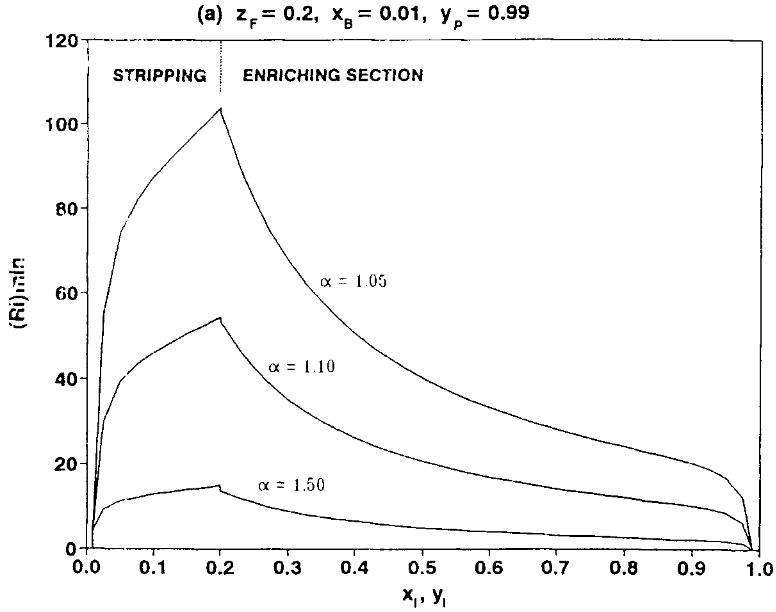


FIG. 5A Plots showing how $(R_i)_{min}$ varies with feed and product compositions as a function of stage compositions for various (constant) values of α. y_P = 0.99. z_F varies from 0.2 to 0.8.

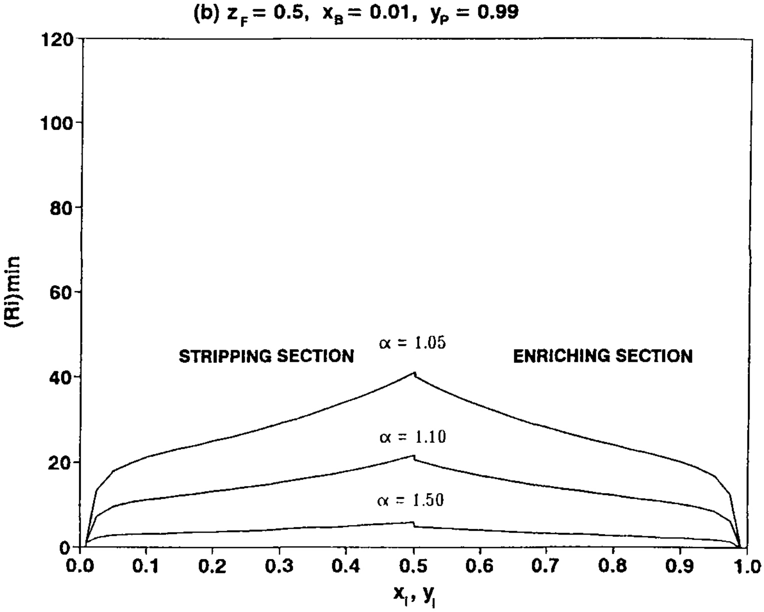
requirement for a partial reboiler or partial condenser at each stage. Again, R_i must be greater than $(R_i)_{min}$ everywhere in the cascade if the cascade is to make the desired separation. Each cascade design would require a different number of stages, and a different total interstage flow.

The total interstage flow is a measure of overall equipment size for the cascade. It is proportional to the number of stages and the interstage flow to each stage. Since R_i need not be as high toward the two product ends of the cascade, it is theoretically possible to reduce the interstage flow with a corresponding reduction in equipment size toward the product ends. A cascade design that assures *minimum* total interstage flow of any cascade design is the *ideal cascade*.

As discussed by Benedict et al. (7), an ideal cascade is one that meets *both* of the following criteria:

1. The two feed streams to each stage in the cascade have the same composition:

$$y_{i-1} = x_{i-1} \tag{13}$$



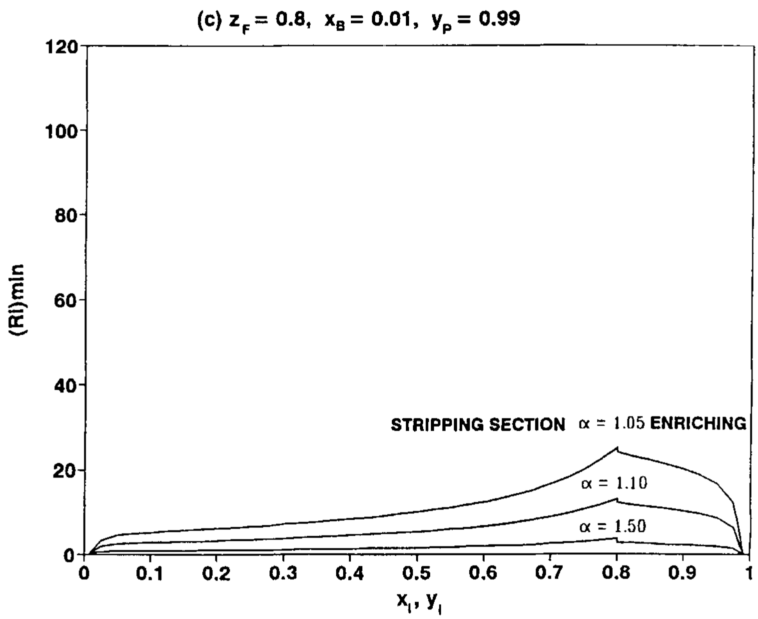


FIG. 5A Continued

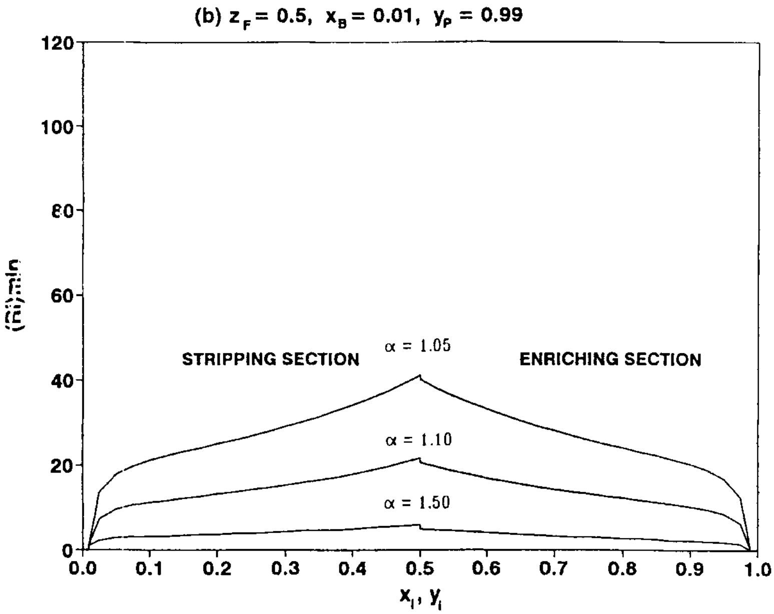


FIG. 5B Plots showing how $(R_i)_{min}$ varies with feed and product compositions as a function of stage compositions for various (constant) values of α. z_F = 0.5, y_P varies from 0.99 to 0.8.

2. The heads separation factor defined by

$$\beta_i = \frac{y_i}{(1 - y_i)} \Bigg/ \frac{z_i}{(1 - z_i)} = \sqrt{\alpha} = \text{constant} \tag{14}$$

for each stage. Here $z_i = y_{i-1} = x_{i+1}$.

It is possible for criterion 1 to be met without meeting 2, but this would not result in minimum total interstage flow, that is, the criterion expressed by Eq. (10) is a necessary but not sufficient condition for an ideal cascade (9).

A *squared-off* cascade is one with several constant recycle sections in it to (possibly) take advantage of smaller recycle ratio requirements toward the product ends of the cascade. This could result in smaller equipment size, since, with the squared-off design, it may be possible to approach the minimum total interstage flow required for an ideal cascade.

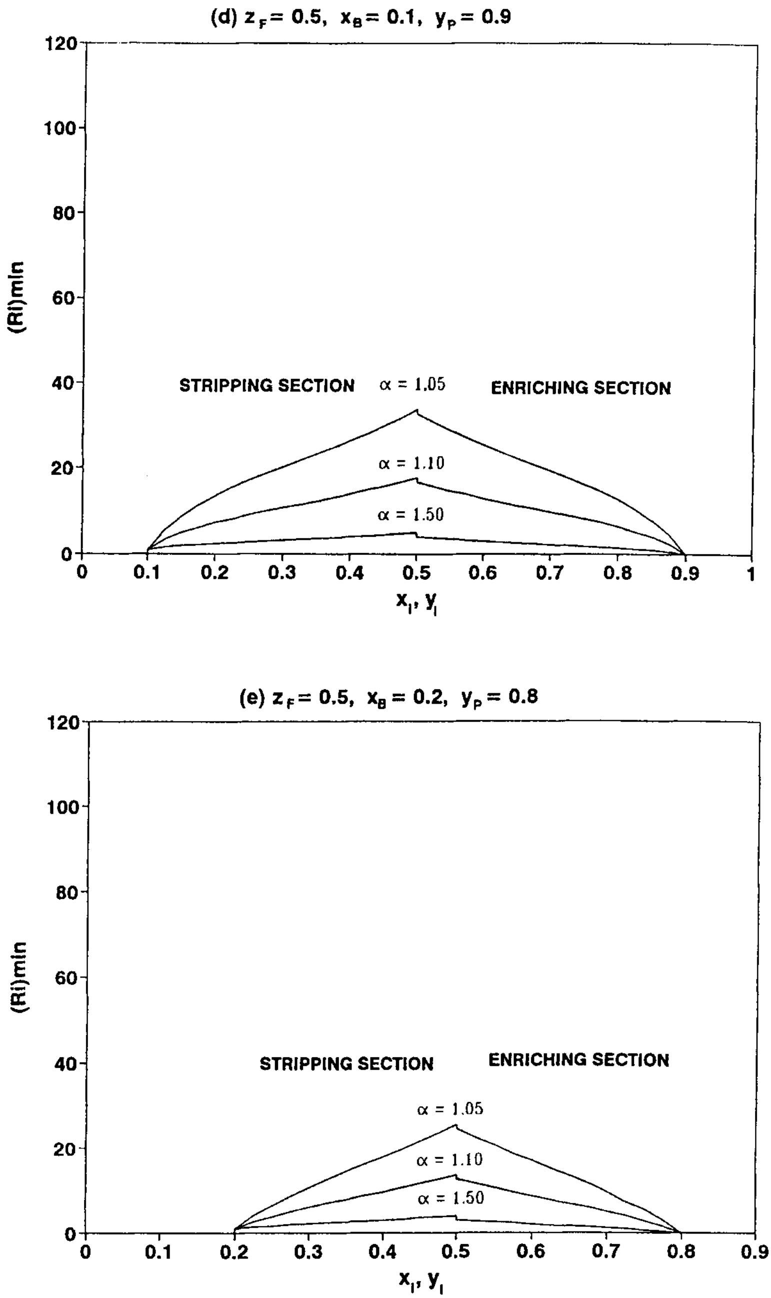


FIG. 5B Continued

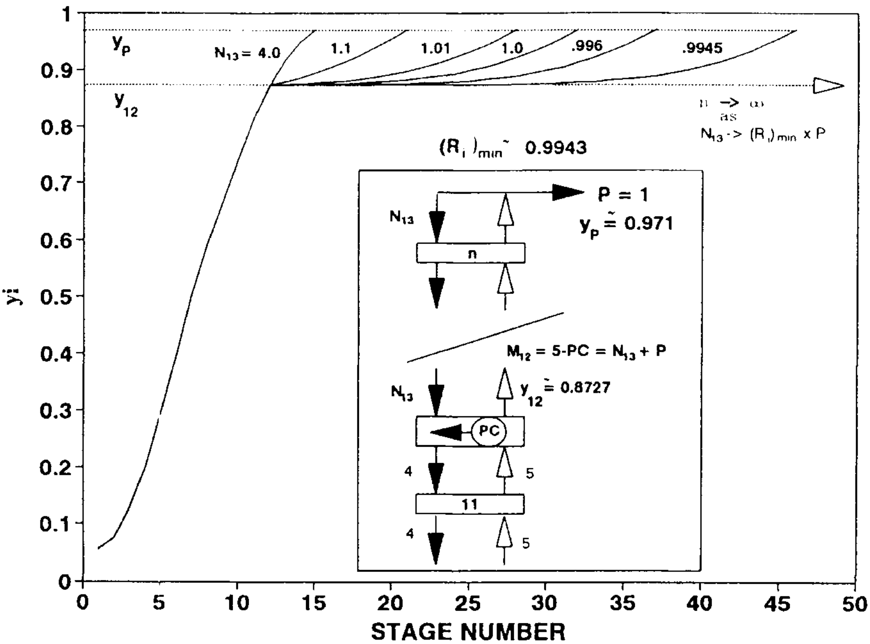


FIG. 6 y_i as a function of stage number for a series of cascade designs with partial condensation on stage 12 in the enriching section of the cascade. Reflux ratio is constant at 4 below stage 12 in the enriching section. $\alpha = 2$, $z_F = 0.5$, flows relative to $P = 1$, PC = partial condensation rate. Assumes constant molal overflow.

Calculations were made for constant recycle, ideal, and squared-off cascades for the Table 2 set of external variables for an arbitrary separation.

Note that the Table 2 set of external variables cannot all be independently specified. For this example, there are six external variables that completely specifies the separation (z_F, F, y_P, P, x_B, B), while two material balance equations can be written that relate these variables:

$$F = P + B$$
$$z_F F = y_P P + x_B B \tag{15}$$

Thus, only *four* of the six variables can be independently specified, while the other two variables are fix by the material balance constraints. For this example z_F, F, y_P, and x_B are specified while P and B are fixed by the material balances. For the chosen "design" values (y_P = 0.99, x_B =

TABLE 2
Assumed External Variables Used for Comparison of Different Cascade Types

$\alpha = 1.5$
$z_F = 0.5$
$F = 2$
$P \approx 1, B \approx 1$
$y_P \geq 0.99$
$x_B \leq 0.01$

0.01), P and B must both equal 1, but since x_B and y_P in reality must be calculated for an integral number of stages, P and B may vary slightly from these "design" values.

In all cases the minimum recycle ratio for the stripping section is put on the same basis as the enriching section by the use of Eq. (9).

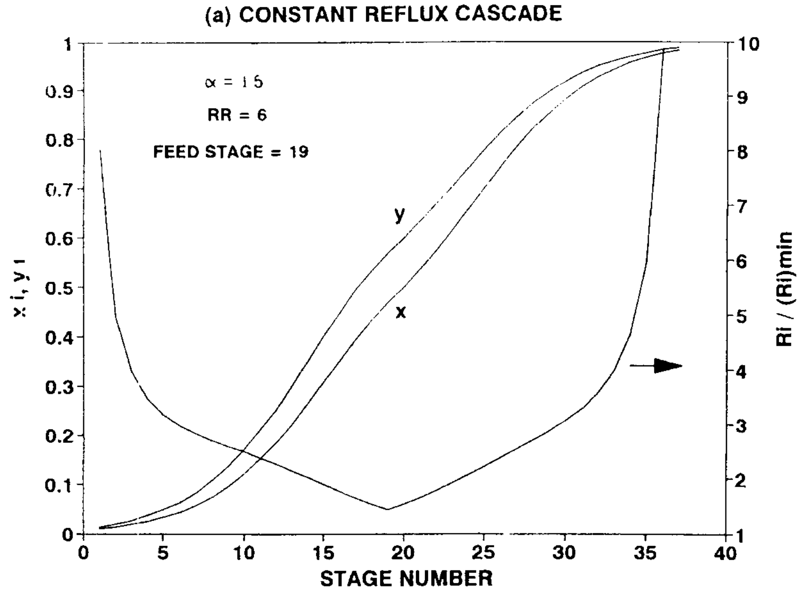
Constant Recycle Cascade

There are very many possible designs for a countercurrent recycle cascade with constant reflux ratio. Reflux ratios varying from a value slightly over $[(R_i)_{min}]_{max}$, to a reflux ratio approaching total reflux will result in a design that will accomplish the desired separation. Each design will require a different number of stages; a great number as $[(R_i)_{min}]_{max}$ is approached, and a minimum as total reflux is approached. The reflux ratio is usually based on economic considerations, but is frequently in the range of 1.1 to 1.5 times $[(R_i)_{min}]_{max}$.

Figure 7(a) presents composition and $R_i/(R_i)_{min}$ as a function of stage number for a constant recycle cascade for the separation described in Table 2, for a reflux ratio = 6.0, which is about 1.4 × $[(R_i)_{min}]_{max}$. For these calculations it was assumed that the feed was saturated, and thus $[N_{j+1}/P] \approx 8.0$ everywhere in the stripping section. A total condenser was assumed to produce reflux to the top stage.

For these conditions, 37 ideal stages would be required to produce $y_P \approx 0.99$, and $x_B \approx 0.01$. As can be seen from Fig. 7(a), $R_i/(R_i)_{min}$ is a minimum (about 1.4) at the feed stage (number 19), and increases to a value of 8 at stage 1 (the reboiler); and to a value of about 9.9 at the top of the cascade, stage 37. Thus from about 1.4 to 9.9 times the minimum recycle is used in this cascade design, depending on stage composition.

Figure 7(b) presents plots of stagewise recycle ratio (R_i) together with $(R_i)_{min}$, since it is interesting to compare various cascade designs in this manner.



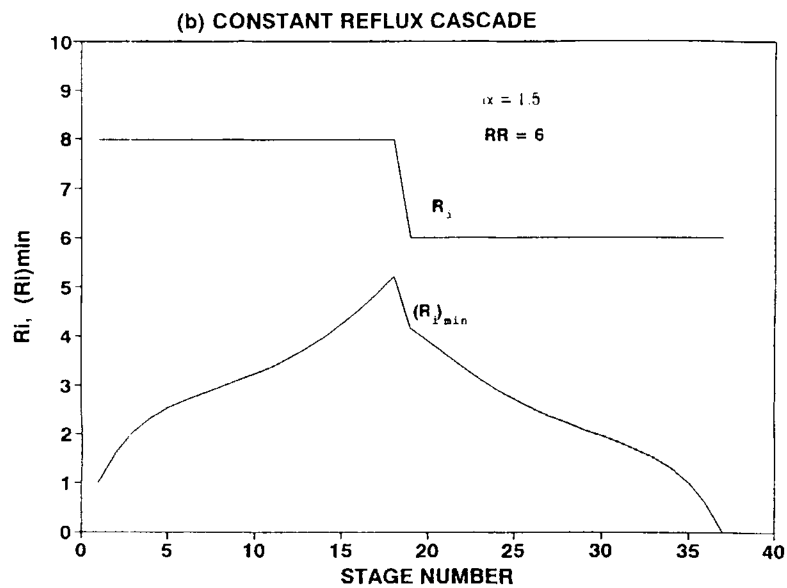
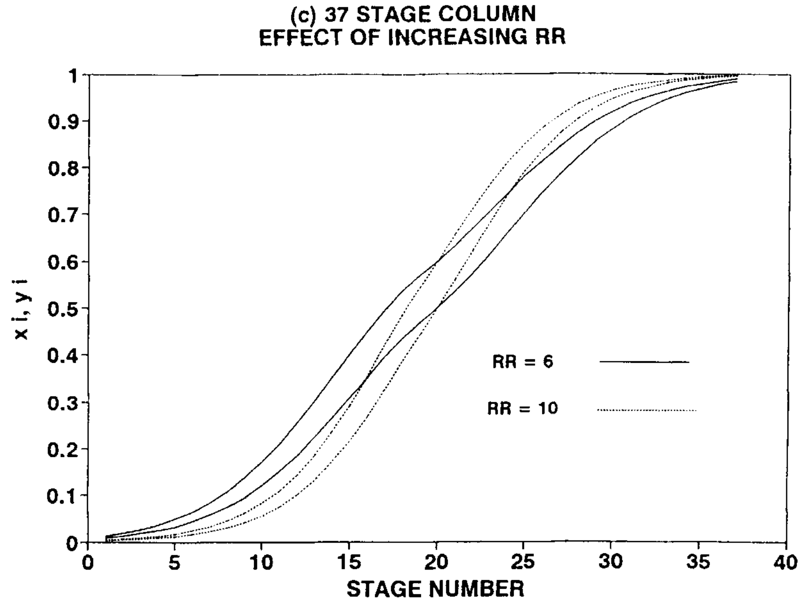


FIG. 7 Constant recycle cascade, (a) and (b) designed to perform the separation of Table 2 with $RR = 6$; (c), (d), (e), and (f) shows the effect of increasing or decreasing RR in the 37 stage column of (a) and (b).



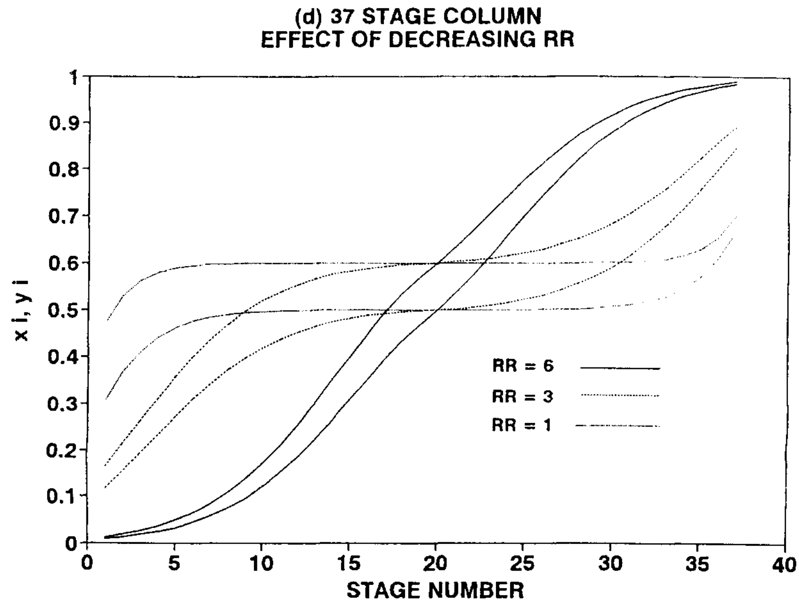


FIG. 7 Continued

(continued)

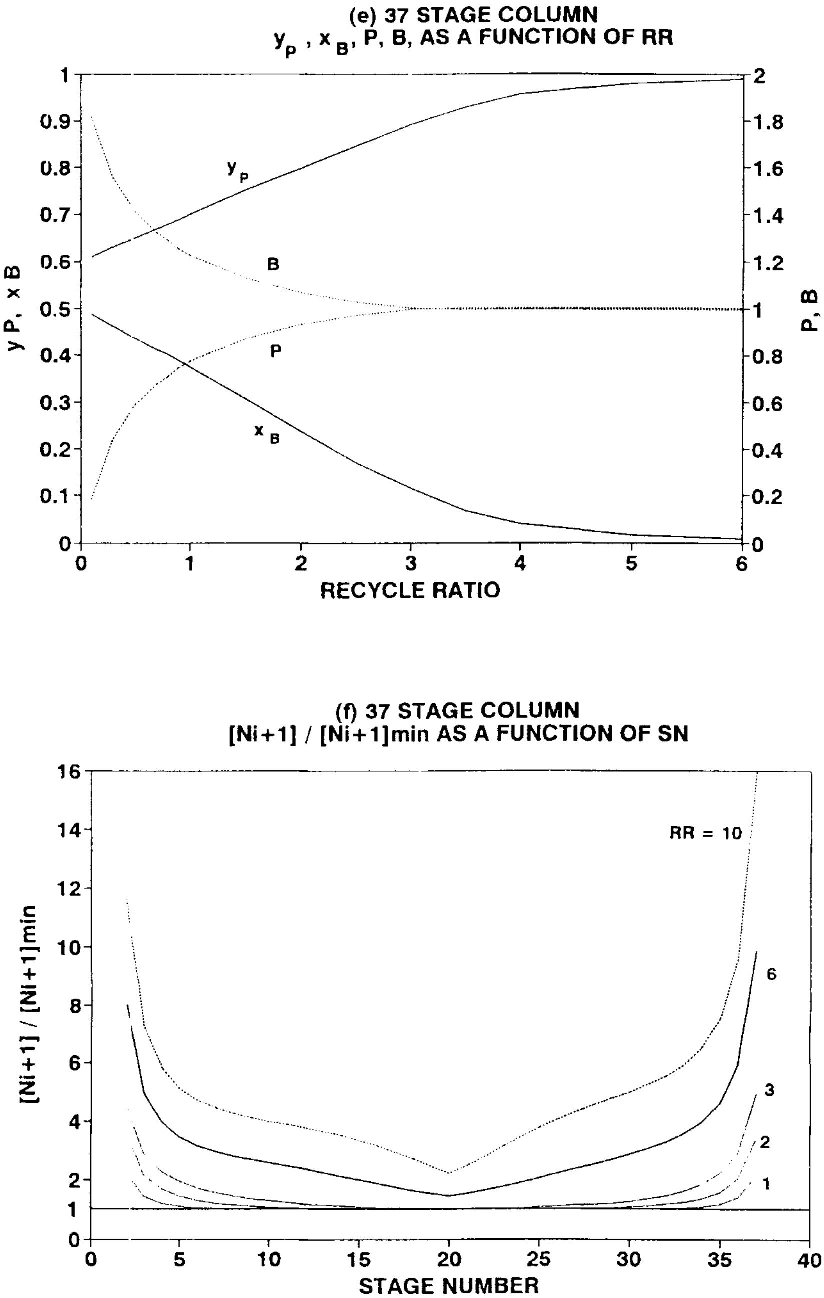


FIG. 7 Continued

Effect of Varying RR for a Fixed Number of Stages. It is also interesting to determine the effect of varying the reflux ratio in a "constant reflux" distillation column containing a fixed number of ideal stripping and enriching stages, since the results clearly illustrates why recycle is required to make a specified separation. These effects are easily calculated with the spreadsheet program by stagewise calculations, again starting at the feed stage. An iterative procedure is conveniently used, assuming the compositions of the two product streams, until the calculated values equal the assumed values and all material balances are satisfied within a convergence criteria. It is convenient to use the previous example as a basis for the comparison.

As shown in the previous example, 37 ideal stages are required to produce $P = 1$, $y_P \approx .99$, $B = 1$, $x_B \approx .01$, with $F = 2$, $z_F = .5$, using $RR = 6$ when $\alpha = 1.5$. This 37-stage cascade contains 19 enriching and 18 stripping stage; that is, the feed stage is number 19. Calculations were made for several *RR* different from the design value. The results were as expected from intuition: the separation is slightly increased for $RR > 6$, while it decreased for $RR < 6$. The results of the calculations are summarized in Figs. 7(c), (d), (e), and (f).

Figure 7(c), which presents x_i and y_i as a function of stage number, shows the effects of increasing *RR* from 6 to 10. As can be seen, a slight increase in y_P and a slight decrease in x_B is achieved, with the slope of the y_i and x_i curves increasing in the "center" portion of the cascade, with the curves becoming more "sigmoid" in shape. Of course, the increased separation is limited for this case, since the column was designed for a high separation with $RR = 6$.

Figure 7(d) presents the same information when *RR* is decreased below the design value of 6. As can be seen, for this case, as *RR* is reduced below the design value, a "pinch" zone develops on both sides of the feed stage [where $(R_i)_{min}$ are the highest], with the zone becoming larger and larger (i.e., covering more and more stages) for the smaller values of *RR*. As a result of this "pinch," x_B must increase and y_P must decrease because less and less separation is occurring on the stages in the pinch zone. In addition, as *RR* is reduced below a certain level, *P* must decrease, and *B* must increase since there is not enough material traveling up the column (the heads stream rate) to produce $P = 1$ *and* provide the specified recycle as a consequence of material balance considerations. Note that actual reflux rate is reduced as *P* decreases.

Figure 7(e) shows how y_P, P, x_B, and B must change with reduced *RR*. As can be seen for this example, reducing the *RR* from 6 to 4 has only a small effect on the separation that is occurring in the cascade, but as *RR* is reduced below about 4 the degradation of the separation is more pronounced. At RR below about 3, y_P has reduced, and x_B has increased to values such that material balance constraints require that P decrease from the desired design value of 1, while B must increase. And, as indicated in the figure, in the limit as $RR \rightarrow 0$, the 37 stage column will approach the behavior of a single stage with $y_P \rightarrow 0.6$, $x_B \rightarrow 0.5$ (while $P \rightarrow 0$, and $B \rightarrow 2$). This behavior is, of course, specific for the assumption that the feed stream is saturated liquid. If the feed were saturated vapor, then in the limit $y_P \rightarrow 0.5$, $x_B \rightarrow 0.4$, while $P \rightarrow 2$, and $B \rightarrow 0$. Also for the above example of saturated liquid feed, when $RR = 0$, there will be no separation, and nothing will happen in the enriching section of the column; that is, the feed will simply flow down the stripping section and exit in the reboiler (stage 1) with no separation occurring.

This example clearly illustrates why recycle is required to make a specified separation in a countercurrent recycle cascade, and why R_i above $(R_i)_{min}$ is necessary throughout the cascade. This is emphasized in Fig. 7(f) which shows $N_{i+1}/[N_{i+1}]_{min}$ for the various RR of this example as a function of stage number. As can be seen, for RR below the design value, a section in the column above and below the feed stage develops where $N_{i+1}/[N_{i+1}]_{min}$ is only slightly greater than 1, and the "pinch zone" referred to above (which becomes larger and more pronounced as RR is decreased) is a direct consequence of this inadequate recycle rate.

Also shown in Fig. 7(f) is the $N_{i+1}/[N_{i+1}]_{min}$ vs stage number curve for the case where RR is increased over the design value. Naturally, the change in the separation curve ($RR = 10$) shown in Fig. 7(c) is a consequence of $N_{i+1}/[N_{i+1}]_{min}$ being greater on all stages for $RR = 10$ compared with $RR = 6$.

The constant recycle design requires one reboiler and one condenser. The consequences of this will be explored below.

Ideal Cascade

As discussed by Benedict et al. (7), the number of ideal stages required for an ideal cascade (N_{IC}) is completely fixed by the criteria expressed by Eqs. (13) and (14), and is equal to

$$N_{IC} = 2 \times N_{TR} - 1 \tag{16}$$

where N_{TR} is the minimum number required at total reflux, as determined by the Fenske-Underwood equation:

$$N_{TR} = \frac{\ln[y_P(1 - x_B)/(1 - y_P)x_B]}{\ln \alpha} \tag{17}$$

For the separation described in Table 2, $N_{TR} = 22.67$ and $N_{IC} = 44.34$. Thus, for these calculations, $N_{IC} = 45$ because a discrete number of stages was assumed.

Note that the no-mix criterion, Eq. (13), cannot be met for stage n in the enriching section of an ideal cascade if a total condenser is used, and so a partial condenser must be assumed at stage n for this design.

The calculations for the ideal cascade are easily made starting at the feed stage, and applying Eqs. (1), (13), and (14) to yield stage compositions as a function of stage number. After stage compositions are determined, material balances yield the required stage heads and tails rates. R_i as a function of stage number is unique for the ideal cascade; only one set of recycle ratios will result in the compositions required for this cascade design, while the number of stages required for an ideal cascade is fixed, as discussed above.

Figures 8(a) and (b) presents compositions R_i, $(R_i)_{\min}$, and $R_i/(R_i)_{\min}$ as a function of stage number for an ideal cascade designed to carry out the separation described in Table 2. As can be seen, 45 ideal stages are required for the ideal cascade to make the specified separation, with $R_i/(R_i)_{\min}$ varying from about 1.8 at stage 1 to somewhat over 2.2 at stage 45, with the variation being described as a sigmoid-shaped curve. Figure 8(b) shows how R_i and $(R_i)_{\min}$ individually vary with stage number, with the maximum occurring at the feed stage, and tapering to smaller values at both ends of the cascade. The diamond-shaped curves for R_i and $(R_i)_{\min}$ vs stage number are characteristic of an ideal cascade, with a relatively large interstage flow required at the feed stage, and a fairly rapid decrease toward the two product ends of the cascade. As mentioned above, the ideal cascade design results in a minimum total interstage flow of any cascade design (7).

This design requires a partial reboiler at each stage in the stripping section, and a partial condenser at each stage in the enriching section.

Squared-Off Cascade

There are virtually an unlimited number of squared-off cascade designs possible to make the separation specified in Table 2, since the number of squared-off sections can vary from 1 (a constant recycle cascade) to where each stage has a different recycle ratio. An ideal cascade is a special case of the latter. From a practical standpoint, the squared-off design should offer some economic advantage with respect to equipment size, with a compromise with respect to complexity of the cascade. Ultimately the feasibility of using the squared-off design should be based on economic considerations, but theoretically it is possible to make any desired separation with it. Only one possible squared-off design will be considered here, to illustrate the concept.

For this illustration a three section design was arbitrarily chosen with $RR \approx 1.4 \times [(R_i)_{\min}]_{\max}$ being used between $y_i \approx 0.2$ and 0.8, $RR = 4$ in

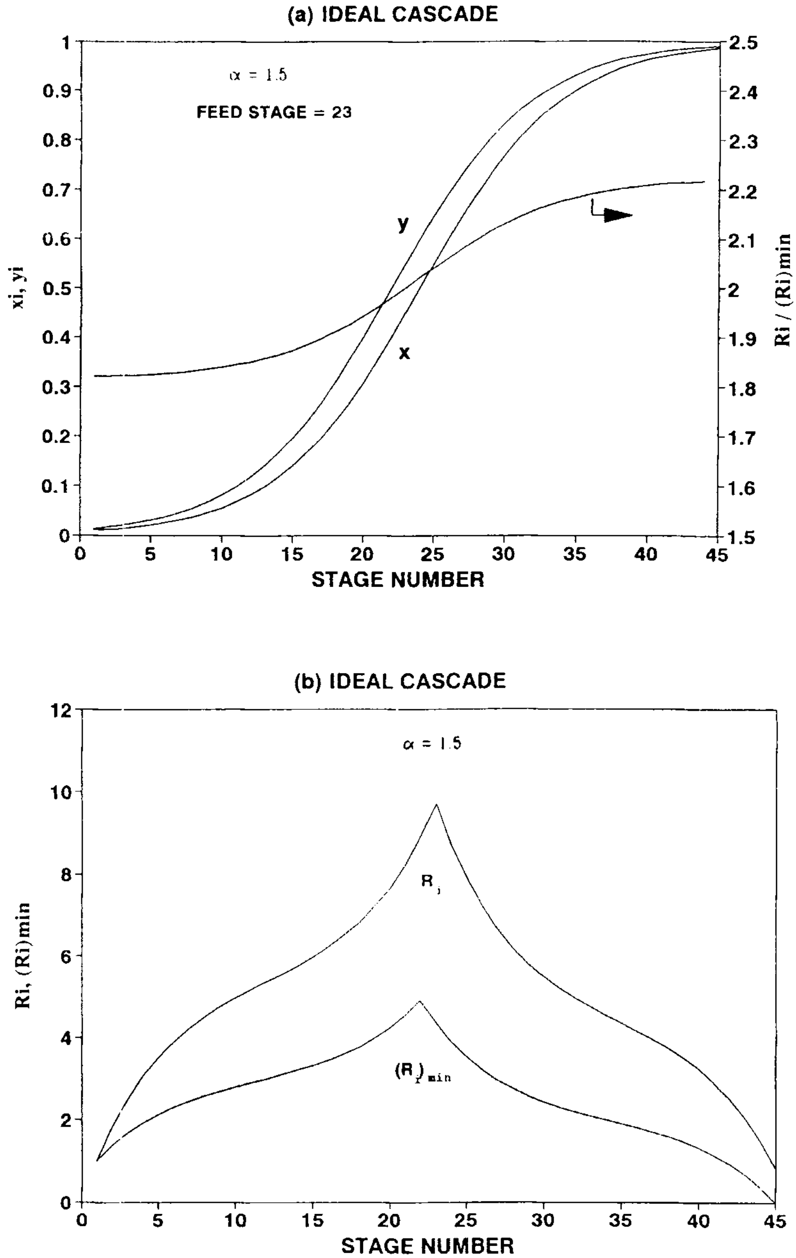
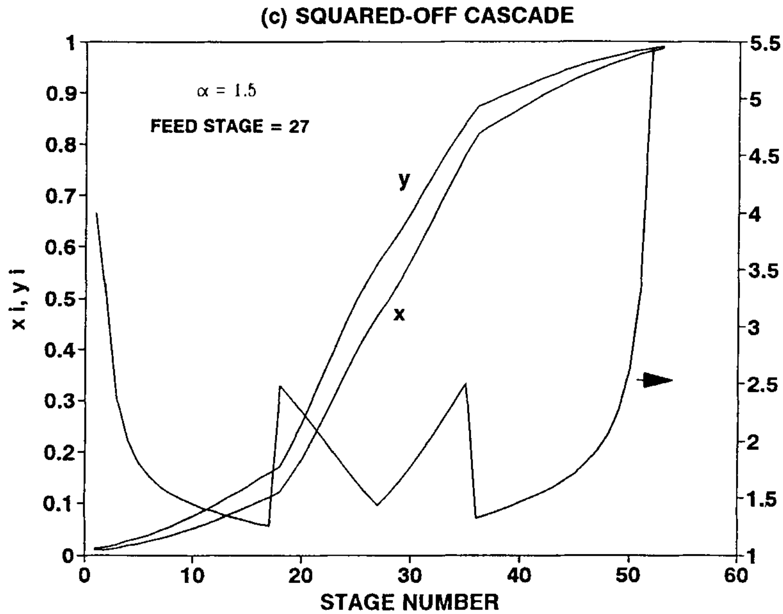


FIG. 8 Composition, recycle ratio, $(R_i)_{min}$, and $R_i/(R_i)_{min}$ as a function of stage number for ideal and squared-off cascades to perform the separation specified in Table 2.



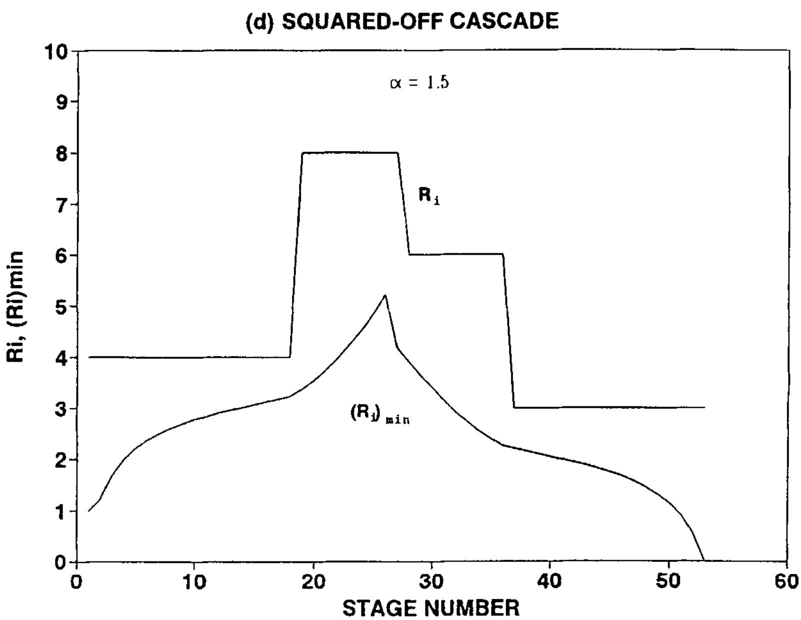


FIG. 8 Continued

TABLE 3
Total Interstage Flow for the Three Cascade Designs. Flow Rates Are Relative to $P = 1$

Design	Heads	Tails	Total	Total/(Total)$_{IC}$
Constant recycle	259	251	510	1.141
Squared-off	249	246	494	1.108
Ideal	223.42	223.42	446.84	1.000

the stripping section below this center section and $RR = 3$ in the enriching section above it. Note that the center squared-off section contains part of both stripping and enriching sections, with $RR = 6$ in the enriching section and $RR = 8$ in the stripping section below the feed stage. This design required 53 total ideal stages to perform the separation, with 17, 18, and 18 stages required in the bottom, center, and top squared-off sections, respectively. The results of calculations are presented in Figs. 8(c) and (d) for comparison with the constant reflux and ideal designs.

This design requires partial condensers at stages 1 and 18, and a partial condenser at stage 36. A total condenser was assumed that provides reflux to the top stage (stage 53), but in fact total condenser duty is distributed between the two (partial) condensers. These partial reboilers and condensers provide for the assumed RR in the three sections of the squared-off cascade.

Total Interstage Flow

The total interstage flow of heads and tails streams within a cascade is a measure of the overall size of the separation plant. For example, the required diameter of a distillation column is usually thought of as being proportional to the interstage (heads) rate. Thus, it is interesting to compare the total interstage flow rates for the three cascade designs discussed above. These values were obtained by summing the heads and tails rates leaving all stages in the cascade. These values are presented in Table 3.

As can be seen from Table 3, the constant recycle and squared-off designs require about 14.1 and 10.8% greater total interstage flow than for the ideal cascade design, which requires an absolute minimum total interstage flow to make the desired separation.

GENERAL DISCUSSION

Figure 5 (which is obtained by plotting Eqs. 7 and 8) shows that minimum recycle ratio depends on α, the desired product compositions, and local (stage) compositions in a complex manner. Since local composition (and possibly α) varies from stage-to-stage, $(R_i)_{min}$ is a *stage-composition* phenomenon, and *each* stage in a distillation column will have a unique $(R_i)_{min}$ associated with it, independent of other stages in the cascade. In addition, the number of ideal stages required for a given separation also depends on α and local flow rates. As shown in Fig. 6, as $(R_i)_{min}$ is approached, more and more stages are required to reach the local composition associated with $(R_i)_{min}$, with the number becoming infinite in the limit. This means that less and less separation is occurring between adjacent stages as the stage composition associated with that $(R_i)_{min}$ is approached, and in the limit, liquid composition leaving adjacent stages will have the same value, that is, separation will cease to occur altogether. Under these conditions, if equilibrium conditions required for the ideal stages are met, the difference in the vapor compositions leaving adjacent stages will also become less and less, with the compositions becoming equal at the limit stage. Although Fig. 6 is an example for a case of $(R_i)_{min}$ at a specific vapor composition leaving a stage in the enriching section of a column, it follows that similar plots would result at any stage composition in either stripping or enriching sections, including the feed point composition. It is convenient to present the minimum recycle ratio equations (Eqs. 7 and 8) in terms of equilibrium vapor compositions for the enriching section and liquid compositions for the stripping section, but several other equivalent forms of these equations can be obtained by use of Eq. (1) (see Appendix 1).

Simple material balance and equilibrium calculations around the stages on both sides of the "pinch point," together with material balances around the top part of the column, show that the material balance and equilibrium requirements can be met with values of N_{i+1} below $[N_{i+1}]_{min}$. However, the results of the calculations for liquid rates below $[N_{i+1}]_{min}$ are meaningless since they yield composition values below the pinch-point composition, which would require an infinite number of stages to reach, and hence is unobtainable. Or, looking at it in terms of a McCabe–Thiele construction, these compositions would be outside the equilibrium x, y curve, and no point on the operating line can lie above the equilibrium curve (3, p. 329).

Equations (7) and (8) result from a consideration of overall material balance requirements for adjacent stages in stripping and enriching sections. Also implicit in the derivation of these equations is that the compositions of heads and tails streams leaving each stage be related by Eq. (1). However, these equations apply for any local value of α, including the case for a nonideal system where α varies with composition.

The minimum interstage flow is given by Eqs. (7) and (8) for both differential and stage types of processes (6). Robinson and Gilliland (5, p. 176) present an equation for $[N_{i+1}/P]_{min}$ that is equivalent to Eq. (7) but they never use it because the equations "involve so many factors that it is usually easier to obtain minimum reflux ratio graphically."

As shown in Fig. 5, $(R_i)_{min}$ is greatest at the feed point (the point furthest from the desired product compositions) and tapers to smaller and smaller values as product compositions are approached. Typically, a constant recycle countercurrent recycle cascade is designed for a reflux ratio from about 1.1 to 1.5 × $[(R_i)_{min}]_{max}$ for economic reasons. It follows that R_i will be greater than the particular design value of $(R_i)_{min}$ everywhere else in the cascade for this constant reflux ratio design. The constant reflux ratio design will result in the requirements for a different number of stages, and a different total interstage flow for each assumed reflux ratio. Total interstage flow depends on flow rates to each stage and the number of stages, and is a measure of equipment size, while the maximum total interstage flow is a measure of energy input required to make the separation using that particular design. The "best" reflux ratio to use can be chosen based on economic considerations which reflects the cost for equipment and energy. The case considered here for illustration purposes (Figs. 7a and b), arbitrarily assumed a reflux ratio of 6, which is about 1.44 × $[(R_i)_{min}]_{max}$.

The calculations made to determine the effects of reducing *RR* in a cascade containing a fixed number of ideal stages clearly illustrates why recycle is required to carry out a specified separation in a CRC. As the *RR* is reduced below the design value, a section in the "center" portion of the cascade develops [where $(R_i)_{min}$ is the highest] where little separation is occurring on the individual stages because the $(R_i)_{min}$ values are being approached. As a result, the overall separation within the cascade must decrease. In the limit, as $RR \rightarrow 0$, as shown in Fig. 7(e), the cascade behavior approaches the behavior of a single stage with zero cut. Finally, for the case considered, no separation will occur with $RR = 0$.

The ideal cascade is unique: only one set of R_i vs stage number will result in an ideal cascade. The ideal cascade design minimizes the total interstage flow required for the separation, and hence it also minimizes the overall equipment size required for the specified separation. As can be seen from Fig. 8(b), the ideal cascade design requires a different recycle ratio at each stage which varies from about 1.8 to 2.2 × $(R_i)_{min}$ with a value of 2.0 × $(R_i)_{min}$ required at the feed composition. The ideal cascade requires (2 × NTR − 1) stages which can be greater than for a constant recycle design, depending on the (constant) reflux ratio assumed. A squared-off cascade can be designed to approach the requirements of an ideal cascade with respect to total interstage flow. The three section SOC design used in this study for illustration purposes was arbitrary both in the number of sections and reflux ratios used in the different sections, with no attempt at optimization. In this arbitrary design the center section contained parts of both stripping and enriching sections, with the other two sections being the rest of the stripping and enriching sections of the cascade. Even with this arbitrary design, the total interstage flow requirements were reduced over the constant reflux case.

Design for Varying Recycle Ratio: Partial Condensers and Reboilers

To our knowledge, stage partial condensers and reboilers have not been routinely used to vary interstage flow throughout a distillation cascade, but conceptually it is certainly theoretically possible. It would take a tray design which is both a vapor–liquid contacting and heat transfer device. This could be accomplished, for example, by circulating heating or cooling media through conduits incorporated within the tray structure, or via a pump around. It would probably be somewhat more difficult to accomplish heat transfer within a packed bed column at the required locations, but certainly it *could* be done.

Along with this, it should be noted that the feed condition with respect to enthalpy (i.e., the q line) can produce partial condensation or reboil at the feed stage.

Energy Requirements to Drive the Distillation Processes

A detailed study of the thermodynamic efficiency of different distillation cascade designs to make a specified separation is complex and beyond the scope of the present study, but it is interesting to present some generalizations for the different cascade designs with respect to energy input requirements and possible thermodynamic efficiency.

Distillation is classified as a *potentially reversible* process, that is, one in which the net available energy required to carry out a specified separation can theoretically be reduced to the minimum required by thermodynamics.

In ordinary distillation (as described by the constant reflux cascade), the interstage flow is established by adding heat to the reboiler at stage 1 (and possibly the feed stream) to produce the up-flowing vapor necessary for the CRC operation. Heat is removed in the condenser at the top of the column, and part of the condensed material is returned to the top of the column as reflux, thus establishing the necessary downflow of liquid. The more volatile (lower boiling) component is concentrated in the (top) product as a result of the multistage process, while the heavier (higher boiling) component is concentrated in the bottoms product stream. As a result, there is a "flow" of heat energy from the higher temperature in the reboiler, where heat is added, to the lower temperature in the condenser, where heat is rejected; there is a temperature gradient in the separation cascade. In an adiabatic column with constant molal overflow (equal heat capacities and latent heats), the stagewise vapor and liquid rates will be nearly constant throughout the stripping and enriching sections. The stagewise and overall work of separation must come from this flow of heat, but otherwise the heat is *reused* from stage-to-stage to produce the equilibrium heads and tails streams at each stage, and maintain vapor and liquid flows. Thus, each design for different reflux ratios will require a different heat input, determined by the flow conditions where $(R_i)_{min}$ is the greatest, i.e., at the feed stage.

In a design such as the ideal cascade which requires a different recycle ratio at each stage (tapering from a high value at the feed stage to smaller values at both ends of the cascade), the total heat input requirement is still determined by the flow requirements at the feed stage. To provide for the recycle flow pattern in the ideal cascade, a partial reboiler is required at each stage in the stripping section, and a partial condenser is required at each stage in the enriching section. Since the energy is reused from stage to stage, the total heat input (the sum of the heat input from all of the partial reboilers) must be enough to maintain the flows required at the *feed stage* where the maximum recycle rate is required. As a result of this, more *heat* input would be required to drive the ideal cascade than the constant reflux cascade discussed above, since the CRC was designed for about $1.44 \times [(R_i)_{min}]_{max}$, while the ideal cascade design requires $2.0 \times [(R_i)_{min}]_{max}$ at the feed stage. Thus, about 1.4 times more heat input would be required to drive the ideal cascade relative to the CRC design discussed above. Total heat input required for a squared-off cascade would likewise be determined by the recycle conditions assumed at the feed stage, and the sum of the heat input from all partial reboilers must be enough to produce the maximum recycle assumed in the SOC design. The ideal and SOC designs also require partial condensers in the enriching sections, with one required at each enriching stage in the ideal cascade.

Efficiency

Thermodynamic efficiency for distillation is usually computed by comparing actual net work required to accomplish the separation in the actual cascade compared with the thermodynamic minimum given by

$$\begin{aligned} -W_{min,T} = RT(&P[y_P \ln(y_P) + (1 - y_P)\ln(1 - y_P)] \\ &+ B[x_B \ln(x_B) + (1 - x_B)\ln(1 - x_B)] \\ &- F[z_F \ln(z_F) + (1 - z_F)\ln(1 - z_F)]) \end{aligned} \tag{18}$$

where the actual net work consumption is determined by the change in *available* energy, that is, when energy is introduced in the reboiler at temperature T_R, travels through the column, and is rejected in the condenser at temperature T_c. The overall change in available energy must account for the actual work required to accomplish the separation. Another way of viewing the efficiency is to compare the entropy increase in a thermodynamic ideal distillation compared with the entropy increase due to the flow of the heat through the column from T_R to T_c. One way to decrease the entropy increase is to use a design such that only a small fraction of the total heat added flows through the full temperature gradient. This is the case for the ideal and squared-off designs. The use of partial reboilers in the stripping sections and of partial condensers in the enriching sections reduces the total degradation of energy as it passes through the cascades by reducing the amount of available energy that is added and increasing the amount that is rejected from the system. In a simple example discussed by Benedict (6), the use of one-stage of partial condensation reduced the entropy increase from 1.25 ($F\mathbf{R}$) to 0.655 ($F\mathbf{R}$), where F is the feed rate and **R** is the universal gas constant. For this example, the minimum entropy increase would be 0.325 ($F\mathbf{R}$). However, the reversibility gained by using partial reboilers and/or partial condensers will *not* reduce the total heat duty required to drive the distillation column. This is determined by the flow requirements at the point where R_i must be a maximum (usually at the feed stage). The use of partial condensers or reboilers only reduces the degradation of the energy as it passes through the system, that is, increases the available energy that is rejected from the system. Although proof appears to be lacking, presumably the ideal cascade and an "optimum" squared-off design would require less *available* energy to make the specified separation compared with constant recycle case, although the ideal and squared-off designs may require more heat *input*. In the latter designs, heat is added not only at T_R but also at lower temperatures in the partial reboilers, while heat is rejected at temperatures greater than T_c in the partial condensers. This means that the average temperature of heat addition is lower, and the average temperature of heat rejection is higher, in the ideal and squared-off designs. Means would have to be devised to take advantage of the difference in the amount of available energy that is used in carrying out the separation if *heat* economy is to be realized (2). This is not likely to be practical for normal distillations.

However, because economics is based both on energy requirements and equipment size, some squared-off designs may be economically attractive for some systems because smaller reflux ratios and hence smaller diameter columns can be used toward the two product ends where $(R_i)_{\min}$ is less than at the feed point.

APPENDIX 1. SOME ALTERNATIVE FORMS OF THE MINIMUM RECYCLE EQUATIONS, TOGETHER WITH THE RANGE OF POSSIBLE STAGE COMPOSITIONS WHERE THEY ARE VALID.

Subscript i refers to enriching section; Subscript j refers to stripping section.

$$\left[\frac{N_{i+1}}{P}\right]_{\min} = \frac{y_P[1 + (\alpha_i - 1)x_i] - \alpha_i x_i}{(\alpha_i - 1)(x_i)(1 - x_i)}, \qquad \frac{y_P}{y_P + \alpha_i(1 - y_P)} \geq x_i \geq z_F$$

$$\left[\frac{N_{i+1}}{P}\right]_{\min} = \frac{(y_P - y_i)[y_i + \alpha_i(1 - y_i)]}{(\alpha_i - 1)(y_i)(1 - y_i)}, \qquad y_P \geq y_i \geq \frac{\alpha_i z_F}{1 + (\alpha_i - 1)z_F}$$

$$\left[\frac{N_{j+1}}{B}\right]_{\min} = \frac{\alpha_j x_j - x_B[1 + (\alpha_j - 1)x_j]}{(\alpha_j - 1)(x_j)(1 - x_j)}, \qquad z_F \geq x_j \geq x_B$$

$$\left[\frac{N_{j+1}}{B}\right]_{\min} = \frac{(y_j - x_B)[y_j + \alpha_i(1 - y_j)]}{(\alpha_j - 1)(y_j)(1 - y_j)},$$

$$\frac{\alpha_j z_F}{1 + (\alpha_j - 1)z_F} \geq y_j \geq \frac{\alpha_j x_B}{1 + (\alpha_j - 1)x_B}$$

The range of stage compositions where the equations are valid assumes a cascade of ideal (equilibrium) stages with $z_{F+1} = z_F$, that is, the feed is mixed with N_{F+1} which has a composition identical to the feed. As a result, the different stage compositions can be in the indicated range, depending on α.

NOMENCLATURE

B	rate of bottoms product
CRC	constant recycle cascade
F	feed rate
N	liquid rate leaving a stage, number of stages in a cascade
P	rate of distillate or top product
RR	reflux rate—the rate of condensate returned to the top of the column to produce the (constant) R_i in the cascade
SOC	squared-off cascade
x	mole fraction in liquid stream
y	mole fraction in vapor stream
z	mole fraction in feed stream

Greek Letters

α	relative volatility
β	heads separation factor defined by Eq. (11)

Subscripts

B	bottoms composition
F	feed composition
i	general stage in enriching section, sometimes in stripping section
j	general stage in stripping section
max	maximum minimum stage recycle ratio
min	minimum recycle ratio
P	distillate product composition

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